PHYS 232

Diffraction & Interference of Visible Light & Microwaves

Last Modified January 5, 2018

Introduction

The goal of this experiment is to investigate the properties of electromagnetic (EM) radiation as it passes through narrow slits. By narrow, we mean that the slit width is not too much larger than the wavelength λ of the EM radiation. EM waves consist of mutually perpendicular oscillating electric and magnetic fields (see Fig. 1). Radio waves, microwaves, infra-red radiation, visible light, ultraviolet light, X-rays, and gamma rays are all examples of EM radiation. The only difference between them is the frequency f, and hence wavelength $\lambda = c/f$, of the oscillations of $\vec{E}$ and $\vec{B}$. Figure 2 shows the EM spectrum from low frequency (long wavelength) to high frequency (short wavelength).
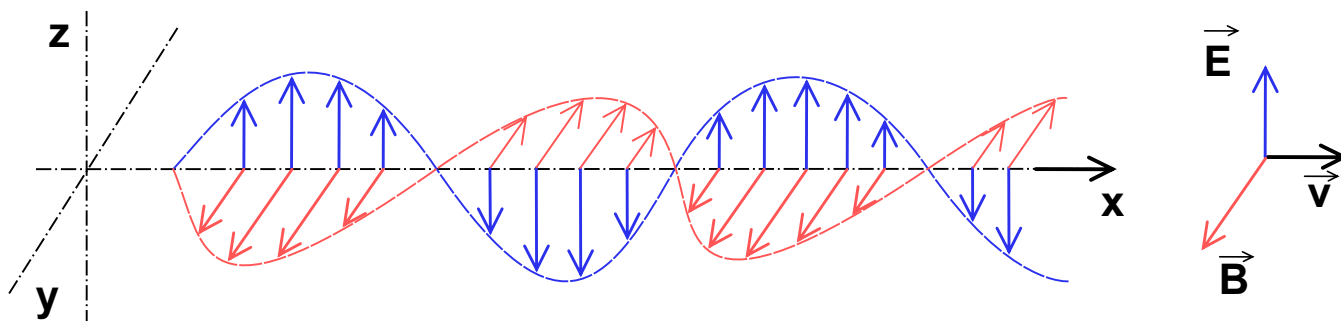

Figure 1: An EM wave propagating in the x-direction. EM waves consist of oscillating electric (blue) and magnetic (red) fields that are perpendicular to one another. EM waves are transverse waves because the electric and magnetic fields are also perpendicular to the direction of propagation. In free space, the speed v of an EM wave is equal to $c = 3.0 \times 10^8$ m/s.
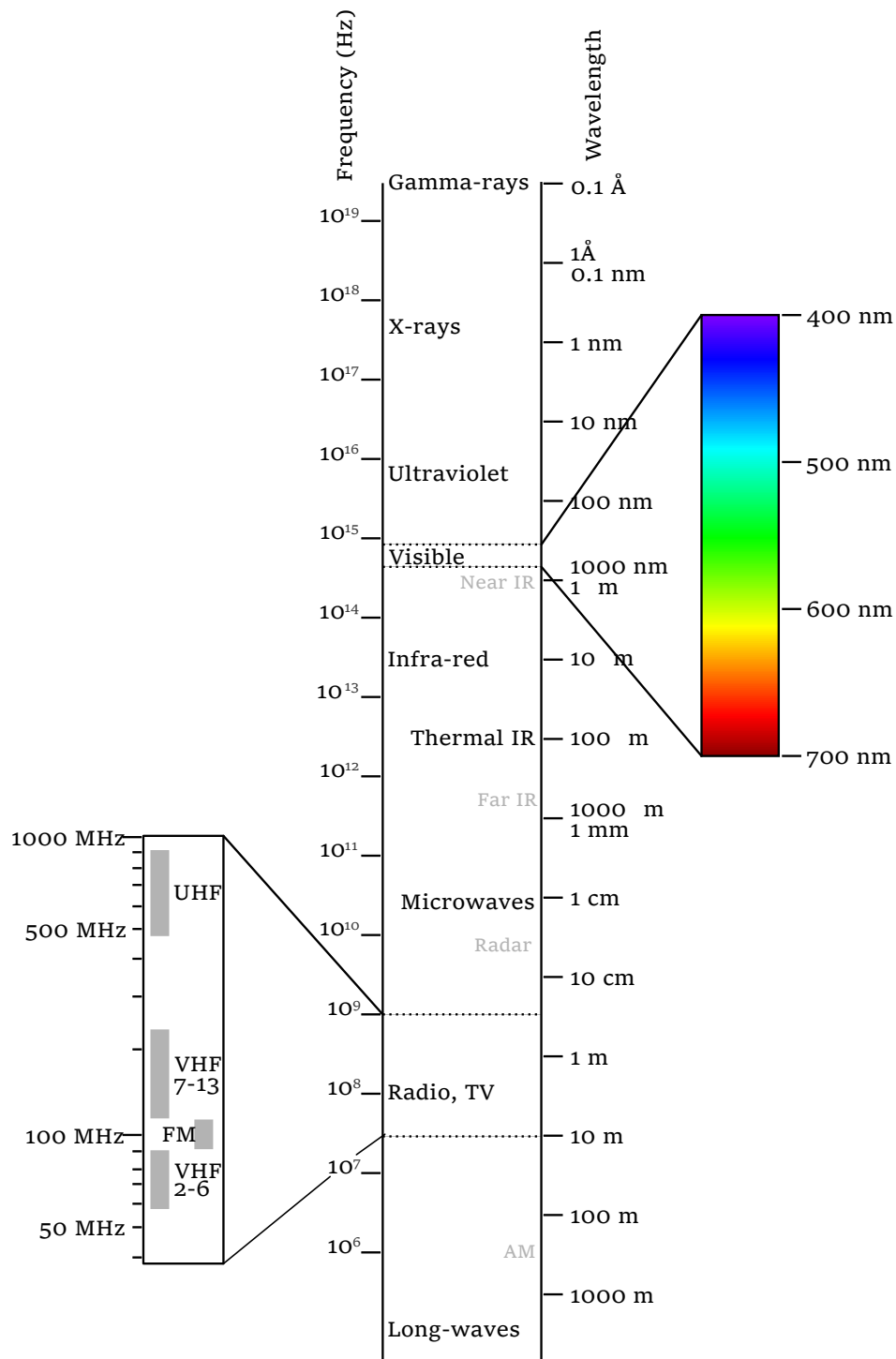
Figure 2: The electromagnetic spectrum. AM radio signals have wavelengths of hundreds of meters and frequencies of hundreds of kHz, whereas Gamma rays have wavelengths of hundredths of nm and frequencies of 10^{19} Hz, spanning more than 13 orders of magnitude.

In our experiments we will use three sources of EM radiation. The first two are red and green light emitted by laser pointers. This EM radiation is part of the visible spectrum with wavelengths of approximately 530 nm (green) and 680 nm (red). The third source that will use a microwave feed horn antenna. The microwave source operates at a frequency of 10 GHz which corresponds to a free-space wavelength of 3 cm, more than four orders of magnitude larger than that of visible light. It is remarkable that we can observe the same type of physical phenomena from EM sources that, at first glance, do not seem to be obviously related. After all, the visible part of the EM spectrum is the part that we can "see" whereas microwaves are used to heat food and transmit/receive cell phone and WiFi signals.

Huygens' Principle

The Huygens (or Huygens-Fresnel) principle offers a useful way to intuitively understand some of the phenomena that we will investigate experimentally. Huygens' principle states that:

> *Every unobstructed point on a wavefront will act a source of secondary spherical waves.*
> *The new wavefront is the surface tangent to all the secondary spherical waves.*

For example, as in Fig. 3, if one has a plane wave (i.e. a planar wavefront), then considering a series of spherical waves originating from the first planar wavefront leads to the formation of a second planar wavefront.
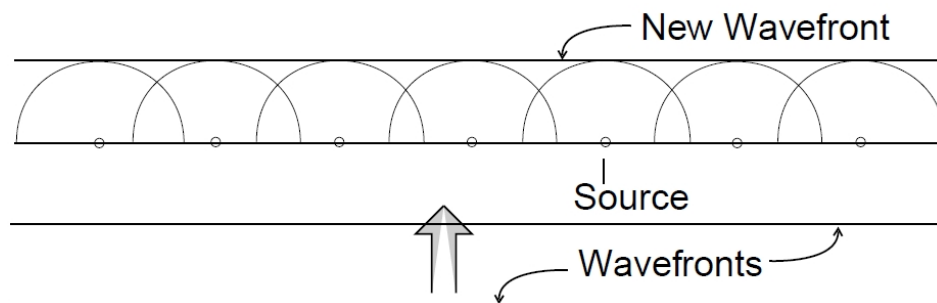

Figure 3: Huygens' principle and plane waves.

As Fig. 4 shows, Huygens' principle can also be used to show that a point source, like a candle, leads to the propagation of spherical wavefronts.
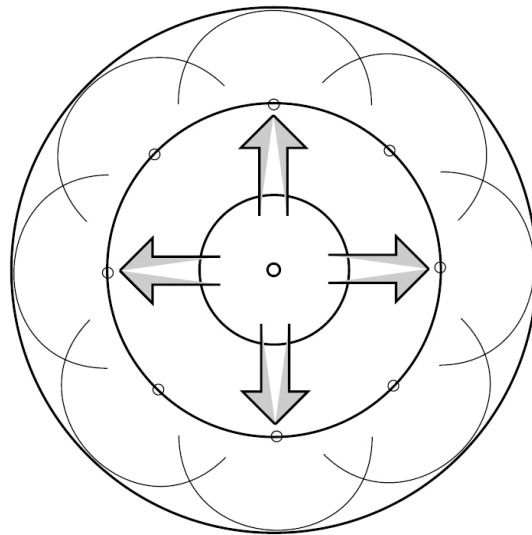

Figure 4: Huygens' principle and spherical waves.

What happens if we apply these same ideas to a plane wave incident on a slit? The geometry of the problem is shown in Fig. 5.
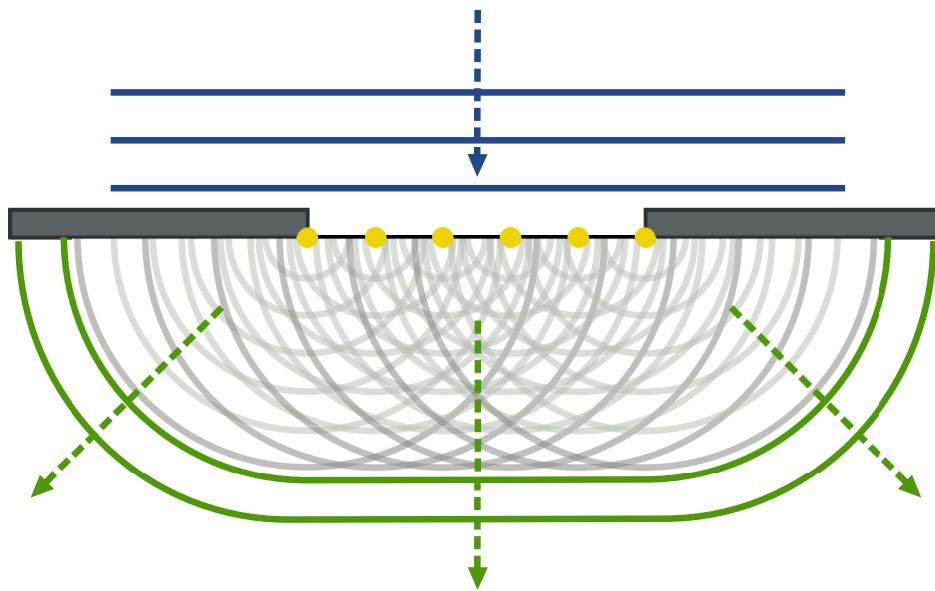

Figure 5: Huygens' principle and plane waves incident on a slit.

Huygens' principle shows that spherical waves that originate near the edges of the slit result in secondary wavefronts are no longer planar. In fact, the principle predicts that, if a screen is placed below the slit, a spot that is greater in extent than the slit will be illuminated. In other words, the light transmitted through the slit bends around the edges.

Superposition & Interference

If you actually send light through a narrow slit, as you will do in this experiment, you will indeed see an extended bright spot illuminating the screen behind the slit. However, that blob will have additional structure. There will be bright regions of varying intensity and also narrow dark spots. To understand the origin of these dark spots requires that we discuss superposition and interference.

The superposition principle states that:

> *For all linear systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually.*

For our purposes, think of the stimuli as individual light rays (oscillating EM fields) converging on a single point. If two rays, of the same frequency, simultaneously arrive at a point *and* they are in phase, then their amplitudes will add *constructively* producing the effect that one would get from a single wave with larger amplitude. Superposition of waves leading to constructive interference is shown on the left-hand side of Fig. 6. On the other hand, if the waves arrive out of phase, their instantaneous amplitudes cancel resulting in destructive interference as shown on the right-hand side of the figure. Notice that the condition for perfect destructive interference is that the two waves be π radians out of phase. Put another way, the two waves are misaligned by half a wavelength.
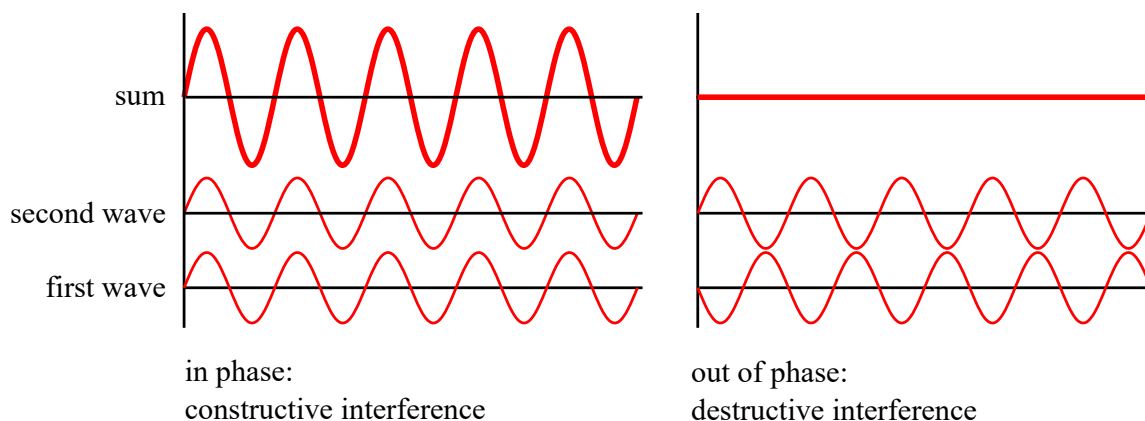

Figure 6: Superposition principle and constructive and destructive interference.

Single-Slit Diffraction

Let's now attempt to combine Huygens' principle, the superposition principle, and the idea of interference to build a more complete picture of the *diffraction* pattern that is expected when EM radiation is passed through a narrow slit. The geometry of the problem is shown in Fig. 7 The points within the slit of width a represent point sources that emit spherical wavefronts (Huygens' principle). If we consider rays that extend from these sources to a point on a far away screen, then all of the rays are approximately parallel and make the same angle θ with respect to a line that is perpendicular to the slit opening. Consider the ray originating from the centre of the slit and a ray originating from the bottom of the slit. The ray at the bottom has to travel an extra distance $\Delta L = (a/2)\sin\theta$ to arrive at the distant point P on the screen. That means there will be a phase difference $\Delta\phi = 2\pi\Delta L/\lambda$ between the two rays at P. This phase difference can lead to constructive or destructive interference depending on the value of $\Delta\phi$. If we want to calculate the positions of the dark spots, then we require:

$$\Delta\phi = 2\pi\left(\frac{a}{2\lambda}\sin\theta_m\right) = \pi m \tag{1}$$

where m is an integer not equal to zero ($m = \pm1, \pm2, \pm3, \dots$) and θ_m locates the mth dark fringe. This condition can be re-expressed as:

$$a\sin\theta_m = m\lambda \qquad m = \pm1, \pm2, \pm3, \dots \qquad \text{(destructive interference)} \tag{2}$$
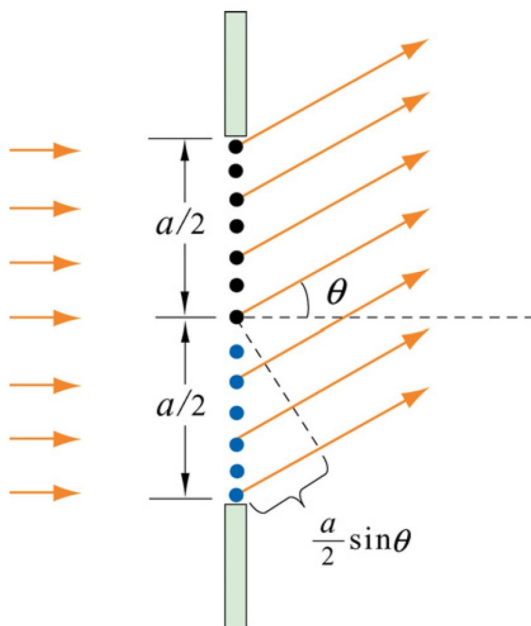

Figure 7: Single-slit diffraction.

We've calculated the condition for destructive interference of the bottom and middle rays in the figure. What about the others? Pick any of the rays originating from within the slit, the superposition of this ray and another one that is either $a/2$ above or below the first will result in destructive interference. Therefore, Eq. 2 gives use a method to calculate the positions of the dark spots within single-slit diffraction patterns.

The supplemental material provided on the course website is very well written and you are encouraged to study it. In that material, the dependence of the diffraction pattern's intensity I on the angle θ is calculated. The result is given by equation 14.6.16 in that document.

Double-Slit Interference

Next, we imagine illuminating two narrow slits (S_1 and S_2) separated by a distance d with a plane wave. The scenario is shown in Fig. 8. In this figure, imagine that $L \gg d$ such that the paths labelled r, r_1 and r_2 are all approximately parallel and, therefore, all make an angle θ with respect to horizontal. We to answer the following question: Under what conditions do rays originating from S_1 and S_2 meet at point P and *constructively* interfere?
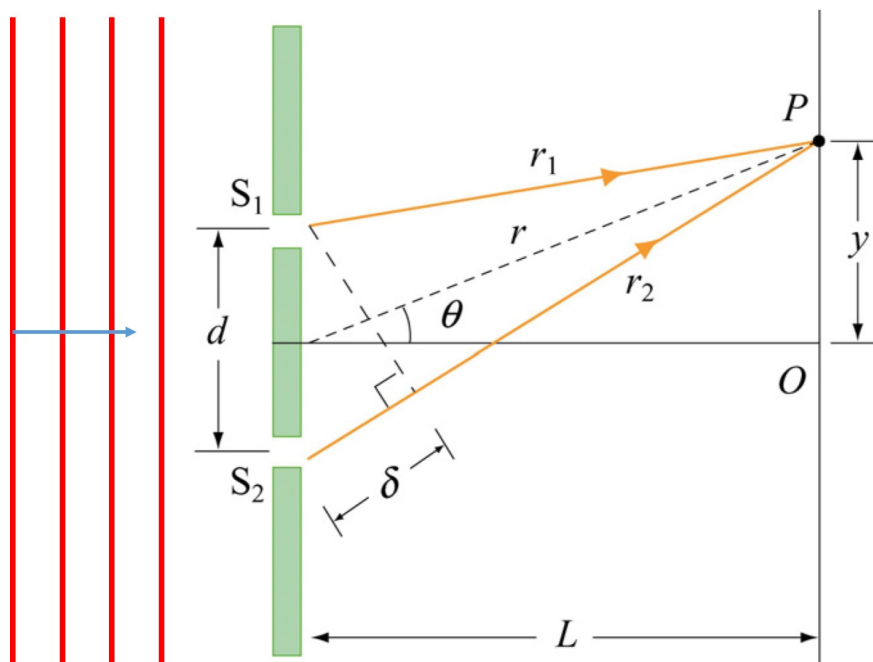

Figure 8: Double-slit diffraction.

The figure clearly shows that, compared to the upper ray, the ray of length r_2 travels an extra distance of $\delta = d \sin \theta$ to arrive at P. For constructive interference we require $\delta = n\lambda$ such that:

$$d \sin \theta_n = n\lambda \qquad n = 0, \pm 1, \pm 2, \ldots \qquad \text{(constructive interference)} \tag{3}$$

In this case, n can be zero because it corresponds to $\delta = 0$ which leads to constructive interference.

In the pre-lab exercises you will show that $\sin \theta_n \approx y_n/L$ such that the constructive interference condition can be expressed as:

$$y_n \approx \left(\lambda \frac{L}{d} \right) n \tag{4}$$

Therefore, if one experimentally measures the positions y_n of the bright spots and plots them as a function of n, a straight line with slope $\lambda L/d$ is expected.

Finally, let's imagine the intensity pattern that we would expect to get on a screen placed far $(L \gg d)$ from the two slits. First, because we have slits, each with an opening of width a, we expect to get the single-slit diffraction pattern discussed in the previous sections. That is, a section of screen that is much wider than a will be illuminated and there will be dark spots whose positions are determined by Eq. 2. Now, however, there is additional interference determined by the spacing d of the pair of slits. The two-slit constructive interference condition is given by Eq. 4. Because one typically has $d > a$, the spacing between two-slit bright spots is less than the spacing between the single-slit dark fringes. Therefore, if you look very closely at an isolated bright region, you should see a fine sub-structure of bright and dark fringes due to the interference between rays originating from slits S_1 and S_2. Again, the supplemental reading calculates the intensity one expects from the two-slit interference pattern combined with the single-slit diffraction profile. See equation 14.7.1 and figure 14.7.1 in that document.

Polarizers

The polarization of an EM wave refers to the direction of the oscillating electric field. So, for example, the EM wave depicted in Fig. 1 is said to be polarized along the z-direction.

Consider what happens when an EM wave is incident on a flat conductor. In electrostatic equilibrium, the electric field inside a conductor (and at its surface) must be zero. Therefore, good conductors reflect EM waves, but introduce a phase shift of π radians. This phase shift guarantees that the sum of the incident and reflected waves will result in zero net electric field at the conductor's surface. On a microscopic scale, the oscillating electric field of the incident wave causes the conduction electrons in the metal to oscillate. These electrons oscillate at the same frequency as the incoming EM wave. The fields radiated by these oscillating charges make up the reflected EM wave. Of course, if you've ever looked in a mirror, you already know that conductors reflect EM waves.
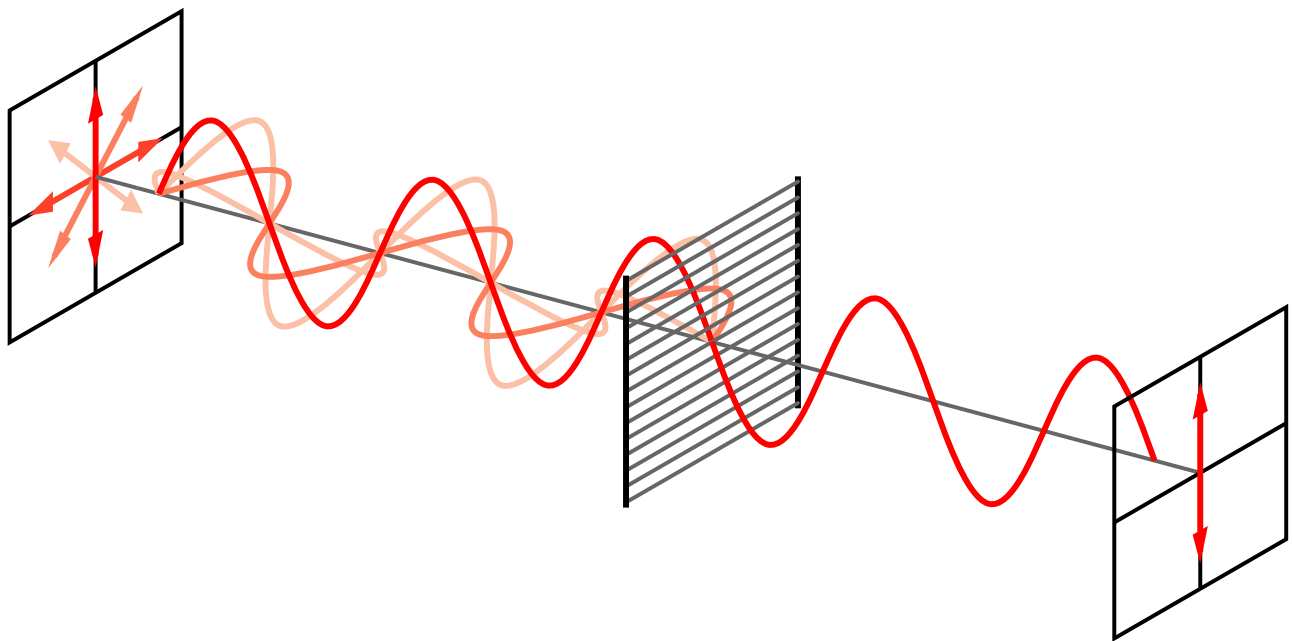

Figure 9: Four EM waves, each with a different polarization, are incident (from left to right) on a wire-grid polarizer. Only the components of the oscillating electric fields that are perpendicular to the wires are transmitted. The horizontal components are reflected by the polarizer.

Consider an EM wave incident on a grid of horizontal wires, as shown in Fig. 9. The sinusoidal lines represent the electric fields of various EM waves. On the left-hand side, four EM waves are incident on the grid of horizontal wires. This grid of conducting wires is called a *polarizer*. Consider the horizontally polarized EM wave incident on the grid of wires. The electric field of this wave easily causes electrons in the wires to oscillate in the horizontal wires and, as a result, the wave is reflected. In other words, none of the horizontally polarized wave is transmitted through the grid.

Next, consider the incident wave that is vertically polarized. It's electric field would tend to cause electrons to oscillate vertically. However, the polarizer is made of thin wires that run horizontally! That is, the motion of the electrons is restricted and, as a result, the vertically polarized EM wave is unaffected by the horizontal wire grid and passes right through it.

What about EM waves that are polarized at some arbitrary angle ϕ with respect to the wires in the polarizer? We can decompose the incident electric field vector $\vec{E}_i$ into components that are parallel and perpendicular to the wires:

$$\vec{E}_i = \vec{E}_\parallel + \vec{E}_\perp = E_0 \left(\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} \right) \tag{5}$$

where the incoming electric field has amplitude E_0 and I've assumed that the wires of the polarizer are parallel to the $\hat{\imath}$-direction. When this wave is incident on the polarizer, the parallel component will be reflected and the perpendicular component will be transmitted. There for the transmitted electric field will be given by:

$$\vec{E}_t = E_0 \sin\phi\,\hat{\jmath}. \tag{6}$$

Recall that the energy density of an electric field is proportional to the square of the electric field. Therefore, if we were to measure the intensity (power per unit area) of the transmitted electric field, it would be proportional to:

$$I_t \propto \left| \vec{E}_t \right|^2 = \vec{E}_t \cdot \vec{E}_t = E_0^2 \sin^2\phi = \frac{1}{2} E_0^2 \left(1 - \cos 2\phi \right). \tag{7}$$

Pre-lab Assignment

Question 1: Consider three equally-spaced slits S_1, S_2, and S_3. The spacing between S_1 and S_2 is d and the spacing between S_2 and S_3 is also d. The three slits are illuminated by by plane waves and the transmitted light is imaged on a screen placed a distance $L \gg d$ from the slits. What is the condition for constructive interference? Give your answer in terms of d, θ (measured from horizontal as in Fig. 8), and λ (the wavelength of the EM radiation). Justify your answer using a neatly drawn figure and a clearly written explanation.

Question 2: What would be the constructive interference condition if we had $N \gg 1$ equally spaced slits? Assume that the spacing between slits is d. Clearly explain your reasoning. These types of devices are called diffraction gratings.

Question 3: Using the geometry of Fig. 8, show that, when $L \gg y$, $\sin\theta$ can be approximated as $\sin\theta \approx y/L$.

Experiment

On the first day you will experiment with visible light passed through single slits, double slits, and diffraction gratings.

Day 1 – single-slit diffraction

First, take the red or green laser pointer and mount it on the optical track such that, when its on, it illuminates the vertical single slit on a slide labelled LH 469 93. Place a screen on the opposite side of the slit. Position the screen far enough away that the dark spots in the diffraction pattern are clearly visible. Stick a piece of masking tape on the screen and mark the positions of the dark spots using a sharp pencil or pen. Make a plot of $\sin\theta_m$ versus m and do a *weighted* linear fit to find its slope. Next, take the laser and determine its wavelength using the visible light spectrometer and Logger Pro. As always, be sure to include estimates of the uncertainty in all of your measurements. Use the slope of your plot and wavelength of the light to determine the width a of the slit and an associated uncertainty.

Repeat the above steps using the other laser pointer (red or green).

Day 1 – double-slit interference

Next, use the translation stage to adjust the position of the LH 469 93 slide such that the laser is incident on the set of two vertical slits. For this measurement, you can use either the red or green laser. Look at the central bright spot. If you look closely, you should be able to see a series of fine dark and bright fringes. You may have to move the screen further away to clearly resolve this structure. Again, place a piece of masking tape on the screen and this time mark the positions of the it bright spots (constructive interference). Plot y_n versus n and fit it to a straight line. You should know λ from the first part of the experiment. Use your slope and value of λ to determine the spacing d of the slits and its uncertainty. You only have to do this part of the experiment using one of the lasers.

Day 1 – diffraction gratings

Finally, get one of the diffraction gratings labelled "191/2". First, hold the grating close to your eye and look at a source of white light. Describe what you see and explain why you see what you do. You don't need to take any measurements, just provide a qualitative discussion in your notebook. Next, place the grating in the path of the laser. You can use either the red or the green laser. Mark the positions of the bright spots. You'll probably need a very large screen and a long piece of masking tape! Plot y_n versus n and determine the line spacing of the grating and, thus calculate the number of lines per centimetre.

Day 2 has two parts, one is double-slit interference using microwaves and the other is polarization of microwaves. These parts can be done in either order. If there are two groups working on this experiment, one group should start with interference while the other does polarization. When both groups are done, make a switch.

Day 2 – double slit interference

For this measurement you are provided with a microwave horn transmitter and receiver. The transmitter produces EM waves at a frequency of 10.525 GHz. The transmitter and receiver are mounted on stands that are positioned with a goniometer. The goniometer allows you to vary the position of the receiver along an arc of fixed radius. Use the magnetic stand and steel plates to construct two uniform slits separated by some distance d. Slit widths of approximately 1.5 cm are recommended. To turn on the microwaves, plug in the transmitted. The receiver is battery operated; switch it on. The receiver outputs a voltage (with selectable range) that is *approximately* proportional to the intensity of the detected microwaves. Measure and plot the intensity as a function $\sin\theta$. Comment on the relative height of the various peaks. Is the behaviour you observe expected? From your plot, determine the $\sin\theta_n$ values of the peaks in intensity (constructive interference) and plot them as a function of n. Using the known slit spacing $d \pm \Delta d$ and the slope of your plot, calculate the wavelength of the microwaves and its uncertainty. Do you get the value you expect?

Day 2 – polarization

The polarization experiment also uses a horn transmitter/receiver pair. For this measurement you will place a polarizer between the microwave source and detector. Turn the transmitter on an measure the intensity of the microwaves transmitted through the polarizer as a function of the polarizer angle. This receiver outputs a DC current that is *approximately* proportional to the detected intensity. To measure the intensity, pass the current through a resistor and measure the voltage across it. Take enough measurements that you can clearly resolve the shape of the intensity versus angle dependence. Do your data exhibit the behaviour predicted by Eq. 7? You don't have to do a fit, just compare the shape of your measured data to the shape of the curve predicted by Eq. 7.